In this study, a new paraconsistent four-valued logic called bi-classical connexive logic (BCC) is introduced as a Gentzen-type sequent calculus. Cut-elimination and completeness theorems for BCC are proved, and it is shown to be decidable. Duality property for BCC is demonstrated as its characteristic property. This property does not hold for typical paraconsistent logics with an implication connective. The same results as those for BCC are also obtained for MBCC, a modal extension of BCC.