We introduce a Gentzen-type sequent calculus $PL$ for a modified extension of Arieli, Avron and Zamansky’s ideal paraconsistent four-valued logic $4CC$. The calculus $PL$, which is also regarded as a paraconsistent four-valued logic, is formalized based on the idea of connexive logic. Theorems for syntactically and semantically embedding $PL$ into a Gentzen-type sequent calculus $LK$ for classical logic and vice versa are proved. The cut-elimination and completeness theorems for $PL$ are obtained via these embedding theorems. Moreover, we introduce an extension $EPL$ of both $PL$ and a Gentzen-type sequent calculus for $4CC$, and show the cut-elimination theorem for $EPL$. The calculus $EPL$ has a novel characteristic property of negative symmetry.