A new logic called linear multilattice logic (LMLn), which is a substructural refinement of Shramko's multilattice logic, is introduced as a Gentzen-type sequent calculus. A phase semantics for LMLn is introduced, and the completeness theorem with respect to this semantics is proved.